one *physical* law of mortality, viz., that no individual known, since the patriarchal ages, shall reach the age of two hundred years; so the period betwixt the birth and death of the oldest individual known would measure the range of that law, which, added to all the different cotemporaneous terms of life, might increase the mean term a unit in some millions! But, admitting many such deaths in extreme old age, from physiological action being worn out, how little would these contribute in establishing a general "physiological law of mortality," as against the great prevailing mortality arising from the accidental causes of death, springing from the circumstances and vicissitudes of life!

On the Origin and Progress of the Calculus of Probabilities. By SAMUEL BROWN, *Actuary of the Guardian Assurance Company, and a Vice-President of the Institute of Actuaries.*

[Read before the Institute, 25th February, 1856, and ordered by the Council to be printed.]

THE application of the doctrine of chances to questions connected with the money values of contingencies depending on human life is now become so common, in this and many other countries, that we may be apt to forget its origin and the process by which it arrived at its present degree of improvement. Although the history of this calculus may be well known to some members of this Institute, there may be others whose time is, perhaps, too occupied to allow of more than the mere study of the calculations and formulæ necessary for the daily routine of a Life Assurance Office, and who may not be aware of the number of other subjects to which, at an early period, the calculus of probabilities was suggested to be applied. I hope, therefore, that in taking a rapid glance at the changes and improvements effected since the first crude ideas on the subject, I may not only be able to excite an increased interest in the science, but induce a further extension of its usefulness.

Poisson remarks that a problem relative to games of chance, proposed to an austere Jansenist by a man of the world, was the origin of a branch of science, now one of the most important in its effects on society. It was in 1654 that the Chevalier de Méré applied to Pascal for a solution of two problems, for which he was unable to find answers. The one was, to ascertain in how many throws one might bet with advantage that two sixes would be

thrown with two dice; the other, to find a rule for dividing the stakes between two players (who were desirous of breaking off an unfinished game), in exact proportion to their relative fortune at the time, and to their chances of winning the remaining stakes. Pascal considered all the possible combinations that could be formed by the simultaneous throw of two dice, and of all the possible changes which might occur in a game of cards, interrupted at any point, and what number of them were in favour of the event for which his solution was required. He then computed the number of cases in which two sixes could be thrown with two dice, and the number of changes which in the actual state of the game of cards would secure to each player, separately, the whole or any part of the stakes, and thus arrived by proportion at the required result. Simple as this method seemed, it was the first attempt to employ mathematics in such subjects—at least, the first that, being closely followed up, led directly to the great discoveries that ensued. Fermat, a magistrate in the Parliament of Toulouse, and a mathematician of great repute in his day, was a friend of Pascal, one with whom he corresponded daily on the subject of his studies, and to whom he freely communicated his doubts and his discoveries. He forwarded to him the solution he had arrived at. The original correspondence is now lost; but it appears clear that in his solution he had merely replied to the questions put to him, and, however ingenious and minute the investigation, it did not lead to ready solutions of other questions of a similar kind. It was Fermat who generalized the solution, and found a rule not merely for ascertaining the value of each player's expectation in the particular case referred to, but at any moment of interrupting the game and between any number of players. This was the next step, and far the most important one, in the science of probabilities. Without it the attempt of Pascal might have remained, like some previous problems and speculations by Galileo and Cardan, in obscurity till a much later period.

The correspondence of Pascal and Fermat was not generally made known at this time, though Pascal, from a letter to one of the learned societies in Paris, in 1654, appears to have entertained the thought of introducing his discovery to the world. Evidently appreciating the importance of it, he speaks of it as a science "qui a scellé l'alliance de la rigueur géométrique et de l'incertitude du sort, et trouvé, dans cette conciliation des contraires, un nom aussi surprenant que mérité, géométrie du hazard—*aleæ geometriæ.*" Pascal, however, about this time met with the acci-

dent which led him to retire altogether from his scientific studies, and devote himself to those religious pursuits of which his celebrated *Provincial Letters* were the fruit. It is said that, as he was riding to the Bridge of Neuilly in a chariot and four, the two foremost horses ran away close to a precipice, down which they rushed into the Seine. The traces broke, and the chariot was left standing on the very brink. This accident so disturbed the brain of Pascal that he ever afterwards imagined there was a precipice on his left hand: he retired to the Abbey of Port Royal, and became almost a religious recluse. Fermat appears to have been indifferent to his discovery, and but little progress was made for nearly half a century.

In 1658 Huyghens, a celebrated geometrician, on the mere rumour of Pascal's questions had published a treatise *De Ludo Aleæ*, which was written in Dutch; but a translation of which into Latin, by Schootens, was appended to his *Exercitationes Mathematicæ*, in 1658. It contained the elements of the new theory, treated with much originality, and with several problems similar to those of Pascal, and the solutions of five curious problems in games of chance; but the subject excited little attention till we arrive at the times of the Grand Pensionary De Wit.

In 1671 this great man, celebrated alike as a statesman and a mathematician of the highest repute, who had already published, in 1650, a work on curves, to which Condorcet refers in terms of eulogy, conceived the design of applying the doctrines of probabilities to the valuation of human life in the question of Government annuities. Collecting the registers of births and deaths in various towns in Holland, he used the results to estimate the true value of a life annuity, in the Report which he prepared on the resolution of the States General, passed 25th April, 1671, to negotiate funds on life annuities. On the 30th July, in the same year, De Wit's report was distributed amongst several of the members. This is a very remarkable document, notwithstanding that it does not show the values of annuities at each age; for not only is the doctrine of chances applied for the first time practically to the solution of one of the most important problems in social or political economy, but the author even notices the effect of selection of the lives of annuitants, and the small difference which is made in the value of an annuity on a young life, whether the table terminates at 77 or a much more advanced age; and concludes, that at the best age the value of a life annuity, in proportion to the redeemable annuity at 25 years' purchase, is really not below but certainly

above 16 years' purchase. A certificate from the Burgomaster Hudde, that, in his opinion, the method employed for the effect sought is perfectly discovered, and that the above result depends upon solid and incontestible mathematical foundations, with a salvo as to the correctness of the calculations, which he appears not to have had time to examine, is appended to the Report. Leibnitz was informed by Huyghens that Hudde had also given his attention to similar subjects, and expressed his regret that the results were not made public. Up to the present time they have escaped discovery; but the original Report of De Wit, after being lost for 180 years, has been recovered by the sagacity and perseverance of my friend Mr. Hendriks, the actuary of the Globe Insurance Company. I need scarcely remind the members of this Institute, that a translation of the entire paper, together with some very valuable and original researches in the history of insurance, by Mr. Hendriks, appeared in the second and third volumes of the *Assurance Magazine*. The circumstances under which the discovery of De Wit's paper was made are too interesting not to bear repetition here. Mr. Hendriks says, "I had for some time been desirous of endeavouring to clear away the veil under which the work in question had been concealed. After fruitless attempts to attain this object here, it occurred as a matter of conjecture that the treatise not included in such collections of De Wit's works as could be referred to was, in all likelihood, written by him for some Government purpose, and consequently might be found in the State archives of Holland, either in a manuscript or printed form. Supplying an esteemed correspondent with some collateral details in support of that view, researches were through his aid made at Amsterdam and the Hague. At length conjecture was justified by the ascertained fact of De Wit's *Treatise on Annuities* having been inserted in the *Resolutions of the States of Holland and West Friesland*, of the year 1671; and being furnished with a transcript of the original in Dutch, and with the more immediate assistance of a French version, the difficulty was at an end." In his subsequent paper, Mr. Hendriks collects various notices on the life of Hudde, and publishes some of his letters; by which we may perceive not only that De Wit and Hudde were jointly prosecuting these inquiries and extending the calculations to two joint lives, and even three, four, five or more lives, but that their attention was directed to the subject in consequence of the recent discoveries in the doctrine of probability by Pascal and Fermat. Notwithstanding the importance of such discoveries, and the very early practical application of

them to public purposes, twenty years elapsed before they drew much attention; and it was not till 1693 that the illustrious Halley, pursuing the same method of investigation, and endeavouring to deduce the law of mortality from the Mortuary Registers of London and Breslau, presented to the Royal Society the first life table, in the form which has ever since been followed, to show the annual movement of a population, the probabilities of survivorship, the average duration of one or more lives, and the money values depending thereon. It is true that in 1661 John Graunt, a merchant in the city, in his *Natural and Political Observations,* made upon the bills of mortality, had, without mathematical knowledge, but by his mere shrewd sense, suggested the proportion of mortality at different ages out of a given number born, and entered into many other curious and original observations; and Sir William Petty, from 1682 to 1687, had also published some sagacious works on the growth of population and rates of mortality; but these did not advance the mathematical doctrines of probability. Even the work of Halley attracted but little notice abroad; and in France, where the first germ of the discovery appeared, the rules and calculations of probability were confined to a few cases of games of chance solved by the learned, or a few experiments to amuse the leisure of the court. Its other great elements of utility were forgotten. More than half a century elapsed before any work of consequence appeared to renew the attention of the public.

James Bernoulli, the eldest of that distinguished family, whose name fills so large a space in the history of mathematics, had long been silently working at a treatise which completely established the calculus of probabilities on the footing of a science. He died in 1705; but, happily, had so far accomplished the great design he had in view, that his work contained the outline of the plan which he intended to fill up, and the greater part finished by his own hands. It was entitled *Ars Conjectandi,* and published at Basle in 1713. In the first part he reprints the tract of Huyghens; in the second he treats of permutations and combinations, the use of which is shown in the third part in various calculations relating to games of chance; and in the fourth he intended to apply the previous reasonings to the events of civil governments and moral and political economy, in which early practical use of the doctrine he was unhappily interrupted by his death. In the subject of the second part he had been already preceded by Leibnitz, in 1666, who thus early in his career, when only at the age of twenty, had published a treatise *De Arte Combinatoria;* and

by Wallis, the English mathematician, who in 1684 had treated of the same subject in his *Treatise of Algebra;* as well as by Father Prestet, in his *Nouveaux Elemens de Mathématiques,* in 1689. But the views of James Bernoulli were so original and profound, that they took precedence of the others, besides having special reference to the theory of chances and analytical investigations connected therewith, which were entirely new. The fourth part, though short, contains the germ of many thoughts and definitions in the art of conjecture, as he terms it. In the very first chapter, he treats of certainty, probability, possibility, the morally certain, the absolutely necessary, and the contingent—regarding truth as a quantity, of which certainty is the whole, and the various shades of probability measured by fractions thereof. There can be little doubt that the publication of this work drew public attention again to the almost forgotten topic, although between the interval of the death of James Bernoulli (in 1705) and the publication of this unfinished work by his nephew, Nicholas Bernoulli (in 1713), another remarkable work had appeared (in 1708), the *Essai d'Analyse sur les Jeux de Hazard,* by Rémond de Montmort. This essay was owing in great measure to the exhortations of Leibnitz, who never lost sight of an opportunity to urge his pupils and friends to pursue the study of this new science, of which, ever since the rumours of De Wit's Report, he had in his letters and in the *Acts of Leipzic* shown his interest by the discussion and solution of various problems. De Montmort, in the preface of his second edition, in 1713, alludes to the labours of all who had preceded him. The preface is consequently a brief but very interesting history of the progress of the science of probabilities to that date. The work does not appear to have come up to the expectation of Leibnitz, and a few analytical errors into which he had fallen in the first edition led to some sharp controversial letters from John and Nicholas Bernoulli and Abraham De Moivre. De Montmort had the good sense to profit by the controversy to improve and correct his own works, and the moral courage to print the letters entire as an addition to it. In the preface he alludes to the labours of Halley; the earliest treatise of De Moivre, in the *Philosophical Transactions,* in 1711; *De Mensura Sortis;* and to a singular memoir, also printed therein, *A Calculation of the Credibility of Human Testimony,* by John Craig; and a work by the same author, *Philosophiæ Christianæ Principia Mathematica,* in which the author seems to propose to convince the Jews and to convert infidels by the aid of geometry and algebra. A much

more important work in the interval mentioned was a Latin thesis, read by N. Bernoulli, at Basle, in 1709, *De Arte Conjectandi in Jure,* in which he proposed to estimate by the rules already laid down by Huyghens the different probabilities of moral and judicial events; as, for instance, the time after which an absent person may be reputed as dead; the premium to be paid for assuring to a young girl a dowry or annuity on the day of her marriage; the relative value of differing testimonies, and the comparative chances of the guilt or innocence of an accused person. It would seem probable that these ideas were suggested by the unfinished work of his uncle, which he afterwards published.

The applications of the science to the numerous and important subjects to which James Bernoulli directed attention appear now to have interested a class of inquirers, who, though not occupying the high places of mathematical science, have probably contributed in no slight degree both to develope in others the highest branches of the study and to give practical utility to what otherwise might remain the mere speculations of philosophers. The collection of facts as to the laws and movement of population, births, deaths and marriages, in different countries, was carried on by men whose services we can now gratefully recognize as leading to the more careful and refined observations of the present day; the results of which, beneficial as they have hitherto been to society, are yet as nothing compared with what remains unexplored in the great field of inquiry which is every day opening up before us. Simpson in England, and Wargentin the Swedish astronomer, were the principal mathematicians of repute who engaged in these researches. The former, in 1740, published his treatise on *The Nature and Laws of Chance;* and, in 1742, *The Doctrine of Annuities.* Short, in his *New Observations on City Bills of Mortality* (1750), and *A Comparative History of the Increase and Decrease of Mankind in England* (1767); Morris, in his *Observations on the past Growth and present State of the City of London* (1751); in Holland, Kerseboom (1738); in Germany, Sussmilch, *Goettliche Ordnung* (1742), and Bielfield, *Institutions Politiques* (1759); and, in France, Deparcieux, in his *Essai sur les Probabilités de la Durée de la Vie Humaine* (1746); and Dupré de Saint Maur, in his *Tables de Mortalité,* inserted in Buffon's *Histoire Naturelle de l'Homme* (1749)—have all contributed their valuable labours to researches which, however defective in many respects, were the first attempts to carry out the ideas suggested by the earlier writers, and induced the necessity of that expansion of the calculus to meet the wants of

society which brought to the subject the talents of the highest mathematicians of a somewhat later age. A long interval has occurred in the history of improvement; but when we see such names as De Moivre, Daniel Bernoulli, D'Alembert, Buffon, Euler, Lagrange, and Laplace, now appear on the stage, we may well allow of a pause for the introduction of such an array of talent.

Of this epoch De Moivre may be considered first as in order of date. Bernoulli had pointed out that the ratio of happening of events, shown by observation in a long series of experiments, approached nearer the true ratio in proportion as the number of experiments was increased. De Moivre furnished a simple and elegant expression for the probability that such ratios are contained within given limits. To express this in figures, long and laborious calculations were often required. De Moivre made use of a theorem of Stirling (*Methodus Differentialis*, &c., 1830) to find the middle term of a binomial raised to a high power, by which he shows that very convergent approximations can be found in many cases which would be otherwise wholly impracticable. He further reduced the labour of calculation by the discovery of recurring infinite series, by means of which the summation of series was greatly facilitated. In the doctrine of life annuities and reversions the formulæ which he suggested carried the subject far beyond Halley by the brevity and elegance of his solutions, and in improving which he occupied at intervals the remainder of his valuable life. His first work on these subjects was in 1711, his last in 1744.

Daniel Bernoulli was the son of John Bernoulli, and nephew of the author of *Ars Conjectandi*. In 1734 a question on the greater or less inclination of the orbit of planets led him to researches as to the probability of the existence of the cause, and to what extent it exceeded the probability that mere chance should produce regular order in any result. This was the first application of analysis to the probability that a certain derangement is the result of a design to produce it. It was Daniel Bernoulli also who introduced into the investigation of relative chances at play the question of the effect of gain or loss on the private fortune of the players. A more practical question—one in which the interests of humanity were concerned—soon drew his attention. Inoculation against the ravages of small-pox, imported from the East, became the subject about that time of violent opposition and disputes in Europe. Bernoulli undertook to show the effect of the proposed measure on the duration of human life. The facts were

few, the conditions to be examined numerous; but his judgment was, that the mean duration of life was prolonged three years by this great discovery. Well may we admire the resources of science, when we see his conclusions almost exactly agreeing with the researches of Duvillard, forty years later. Lastly, Bernoulli introduced into the theory of probabilities the differential calculus, substituting it for the method of combinations, and thus gave additional and powerful aid in the investigation of the most complicated problems.

Buffon, struck by the remarkable speculations of Bernoulli on the subject of the relative gain or loss at play on the private fortunes of the players, published in 1777 (though it was commenced in 1760) a most eloquent and impressive attack on the immorality, the danger, and the vicious principle of Government gaming tables, and contributed perhaps more than any other to their gradual extinction in some countries, and the disrepute to which they have justly fallen in others. The doctrine of chances has seldom been converted to a more honourable or benevolent purpose, and never with greater eloquence or force.

The definitions and claims of this new science found, however, a formidable opponent in D'Alembert, who, reasoning on the treatise of Daniel Bernoulli as to the effects of the introduction of inoculation, contended that the motives of individuals, and their disposition not to submit to the remedy proposed, would alter materially his mathematical conclusions, and suggested that the empire of mathematics ceases when the course of natural events comes to be controlled by the will of man. The simple answer would seem to be, that all observed events depending on free will may be the result of several causes acting together; and that the motives which lead to their occurrence are virtually taken into the estimate of the probability of their recurrence, if the facts themselves have been carefully and accurately observed.

Euler and others scarcely condescended to answer him, but pursued the subject with fresh energy. Besides memoirs on the method of combinations, Euler was the first to suggest the method of taking the mean in the results of observations, which afterwards became of such frequent use in astronomical calculations.

In 1763, however, one of the most important steps in the science was made by Bayes, an English mathematician, who determined the probability that the possibility of the happening of events, as shown by actual experience, should lie within certain limits; and thus gave the first idea of the theory of the probability

of causes, and their future action, deduced from the observation of facts which had actually occurred. In this view he was followed by Laplace, who, in 1774 and 1778, in two memoirs on probabilities, extended the limits of the same inquiry. Lagrange, in 1775, had reduced the theory of recurring series to the differential calculus. Laplace took up the same subject, and, in the course of its application to some of the most complicated problems, established a new theory of the calculus of partial finite differences, which he published in 1776, under the title *Sur l'Intégration des Equations Différentielles aux Différences Finies, et sur leur Usage dans les Théories des Hazards.*

In the meantime the application of the doctrine to the improvement of social institutions had been practically carried out in England by the formation of Annuity and Assurance Companies on pure theoretical principles. In the writings of Dr. Price—especially in his first work, *Observations on Reversionary Payments,* the first edition of which was published in 1769, and the sixth and seventh edited by his nephew, Mr. William Morgan, in 1803 and 1812—attention was drawn to the radically defective schemes of Annuity and Benefit Societies then prevalent, and the true principles of calculation according to the theory of probabilities of human life laid down. Simpson and Dodson preceded him: the former, in 1740, had published his *Laws of Chance.* On the advice of these two mathematicians, the Equitable, in 1762, was the first Society established graduating the premiums according to the probabilities of life at different ages. Dr. Price gave it the benefit of his mathematical knowledge of the subject; and we all know how the astonishing success of that great Society, followed by numerous others to the present day, introduced and established the theory of probabilities as applied to population, on a scale of grandeur and utility which no other nation has witnessed.

Euler, in 1760, in his *Recherches sur la Mortalité et la Multiplication du Genre Humain, et sur les Rentes Viagères,* brought forward the same subject in Germany; and in 1776 Nicholas Fuss, by his *Eclaircissements sur les Etablissements Publics en favour tant des Veuves que des Morts,* prevented the constitution of a Society on an erroneous basis, and popularized the true principles to be followed.

In France, notwithstanding the able works and collections relating to population by Buffon, Moheau, Messance, Neckar, Duséjour, Condorcet, and Laplace, the practical application of the science to social statistics became nullified by the ignorant or

designing persons who recklessly took advantage of the public credulity, and by the ruin of the early Companies, and the misery it entailed on families, succeeded in creating a distrust which, in the matter of life assurance, has scarcely even yet subsided in France. The principal writers on this branch of the subject were St. Cyran (*Calcul des Rentes Viagères*, 1779) and Deparcieux (*Traité des Annuités*, 1781). The French Academy of Sciences endeavoured to extend the theory of probabilities to a new subject, and proposed in 1783 a prize for a "Théorie des Assurances Maritimes"; but it was not till 1787 that they received two papers on the subject, one from Bicquilley and one from Lacroix, which, much altered and corrected, appeared subsequently by both authors as treatises on the calculus of probabilities. Another prize, proposed on the same subject in 1791, met with no success, on account of the more stirring interest of the Revolution.

We now come to the remarkable work of the talented but unfortunate Condorcet, whom that fatal Revolution cut off prematurely in the midst of his mathematical labours and discoveries. From 1781 to 1783 he illustrated by new rules and theorems the science of probabilities as it then stood; but it was in the fifth part of his great work, published in 1784, and still more by his *Essai sur l'Application de l'Analyse à la Probabilité des Décisions rendues à la Pluralité des Voix*, in 1785, that he occupied quite a new and brilliant position in the history of the subject. Commencing with the assumption of assemblies composed of men of equal judgment, knowledge, good faith, and independence of mind, he determines the relative probability of any given number of voters agreeing in a correct judgment. He proceeds to consider such assemblies under various aspects, till he comes to assume them composed of men with qualities all the reverse of the preceding; and extends the theory to the probability of a just judgment of legal tribunals in civil and criminal matters, to the comparative advantages of various modes of election, and, finally, to the varying probabilities of the decision of an assembly increasing or decreasing in number. His conclusions are pure theory, for at that time very few facts collected bore upon the subject; but since his time the statistics of legal tribunals, both in France and in other countries, are yearly collected, and published in a form to admit in part of the application of the rules laid down by Condorcet. It may be easily imagined to what discussion such novel and striking views gave rise; and the illustrations in the earlier part of the work, showing the dependence to be placed on recorded

facts in history, measured by the probability of correct tradition, aroused the men of letters and historians against them. This, however, was but a part of the great subject which Condorcet contemplated. He left behind him at his death the plan of reducing to mathematical laws all the social questions which are acted upon by the free will of man, as well as those which are fixed and immutable in their nature. To this new application of science he gave the appropriate name of "mathématique sociale"; and I need hardly remind some of the members present that of late years such subjects, through the remarkable labours of M. Quetelet, the Royal Astronomer of Belgium, have been constantly advancing in interest.

Up to the time of Laplace all these branches of the science of probabilities had been treated of in different ways and by different methods, which were scattered in various works, memoirs, transactions of learned societies, &c.; but in 1812 appeared the great work of that illustrious mathematician, *Théorie Analytique des Probabilités*, in which all the different methods employed were reduced to one form, and, by aid of the "calculus of generative functions," all the former discoveries were extended, harmonized, and carried to a still greater degree of perfection. There can be no doubt that, to the student desirous at once of perfecting himself in the study of the science of probabilities, this single work contains a complete and harmonious development of the whole subject—to which the *Essai Philosophique des Probabilités*, published by Laplace in 1814, would be an excellent introduction. To complete the history of this subject, as opened up and extended by French writers, we must still add a *Mémoire sur le Calcul des Probabilités appliqué à la Médecine*, by M. Risueno d'Amador, which was published in the *Recueil de l'Académie de Médecine* in 1837, and which gave rise to violent discussions in the medical world as to the propriety of introducing the doctrine of probability to measure the law of disease; and, lastly, a remarkable work by M. Poisson, in 1837, *Recherches sur la Probabilité des Jugements en Matière Criminelle et en Matière Civile*, in which he has, with great power and acuteness, availed himself of the *Comptes Généraux de la Justice Civile et Criminelle en France* (in which facts have been accumulating for several years), to reason not merely on the probability of the correctness of the decision when an accused is declared innocent or guilty, but on the probability whether his condemnation or acquittal by existing laws is, under the circumstances of the country and the age, for the advantage of society and the credit of

its laws. His principal argument is, that though, in a small number of cases, events in the moral world may appear to fluctuate with the caprices of the human will, yet in a large number of cases a marked regularity will be observed, which indicates that, however the causes may be hid, the events themselves happen in conformity with certain laws.

The greatest writer in modern times who has taken up this view of the case is, without doubt, M. Quetelet, of Belgium, one of the most original of thinkers, and who has availed himself of the present advance in statistics to extend the science of probability to questions of moral philosophy and social and political economy which furnish new and striking views of human society. The mere enumeration of his works will show the nature of their contents. They are becoming widely and honourably known in all countries.

1829. Recherches Statistiques sur le Royaume des Pays Bas.

1831. Recherches sur le Penchant au Crime aux Différents Ages.

1835. Essai de Physique Social.

1846. Lettres à S. A. R. le Duc-regnant de Saxe Cobourg, sur la Théorie des Probabilités appliquée aux Sciences Morales et Politiques.

1848. Du Système Social, et des Lois qui le regissent.

The greater part of this historical sketch relates to the great mathematicians of France, because it is there that the science first had birth, and there that it has been carried to a degree of perfection which shows both the ardour and the abilities with which a phalanx of writers of the highest talent have constantly added to its improvement. An admirable Essay by Chs. Gouraud, *Docteur de la Faculté des Lettres de Paris*, in 1848, has afforded me the principal materials for this sketch, which has been completed from the prefaces to the works of several of the French writers alluded to, who trace therein the progress of the improvements to their respective dates.

In England we have had, since the times of Bayes, Simpson, De Moivre, Halley, Stirling, &c., but few original writers on the pure theory of probabilities; but we have carried out the practice in life assurance to a degree which, considering the short period elapsed, astonishes by the grandeur of the interests involved, and the vast amount of benefit which it has conferred upon society.

In 1830 appeared a *Treatise on Probability*, in the Library of Useful Knowledge, by Messrs. Lubbock and Drinkwater, being the first attempt to make the subject popular to the great mass of the

people; and in 1838, Professor De Morgan, in his admirable *Essay on Probabilities, and on their application to Life Contingencies and Insurance Offices,* brought within the reach of the mere arithmetician the rules which, if their demonstration must first be studied in the pages of Laplace, would be confined to mathematicians of the highest order. The student who wishes to consult the higher branches of the subject in English may study with advantage the article on the "theory of probabilities," also by Professor De Morgan, in the *Encyclopædia Metropolitana,* in which the substance of Laplace's great work is briefly given; and a treatise on the same subject by Mr. Galloway, published originally in the *Encyclopædia Britannica,* and afterwards, in 1839, in a separate volume, which contains a summary of the reasonings of Laplace, Condorcet, and Poisson.

Great as has been the progress already made, the application of the theory is still only in its infancy in this country. Assurance on pure theoretical principles is almost entirely confined to the contingencies of human life; whilst the insurance of property against whole or partial loss from any cause, and legal, moral, social and political questions, open up fields of research, and for application of the science, worthy of the high intellectual state of England and of the restless energy of her people.

The Great Powers.

AS the reasonings of the actuary, when extended beyond the mere official routine of a Life Assurance Office, depend upon the proper collection of data relating to population, and the other important elements of the condition and progress of a country, we need make no apology to our readers for presenting to them the following very interesting supplement to the valuable Report of Dr. Farr on the Statistical Congress at Paris in 1855:—

"There are seven great powers in the world.

"England, France, Turkey, and Austria, have existed as great powers for several centuries. Prussia, Russia, and the United States of America, have entered this class within the last hundred years.

"Spain was a great power: she has still a large and not unwarlike population, which has from various causes been left